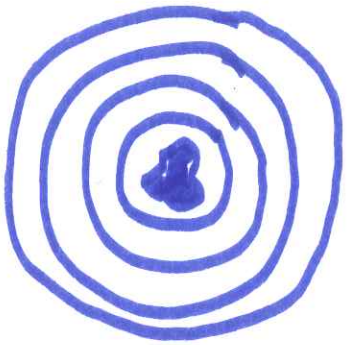
Chapter 7A - Day 1

Related Rates Problems

Ex: Drop a rock in a pond and it creates ripples



The area of the outer ripple/circle is $A = \pi r^2$

but the area and the radius are changing with time.

So, if we take the derivative, we'd take it with respect to t (time)

Recall: Chain Rule:

If y is a function of u and u is a function of x, then y is also a function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex: The area of a circle $A = \pi r^2$ and r depends on t, find a formula for $\frac{dA}{dt}$.

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

now $\frac{dA}{dr} = 2\pi r$ using power rule

so $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

Related Rates Guidelines

1. READ the problem!
2. Identify the variables and draw a picture.
3. Write down what you know and what you are supposed to find.
4. find a relationship (formula) relating your variables.
5. Use the chain rule to take a derivative.
6. Plug in what you know and solve.

Ex: Boyle's Law states that when a gas is compressed at a constant temp, the pressure P and volume V satisfy the equation $PV = C$ where C is a constant. If at a certain instant the volume is 400 cm^3 ($V = 400$), pressure is 200 kPa ($P = 200$), and the pressure is increasing at a rate of 25 kPa/min ($dP/dt = 25$). At what rate is the volume decreasing at this instant? (find $\frac{dV}{dt}$)

$$PV = C$$

take derivative with respect to time.

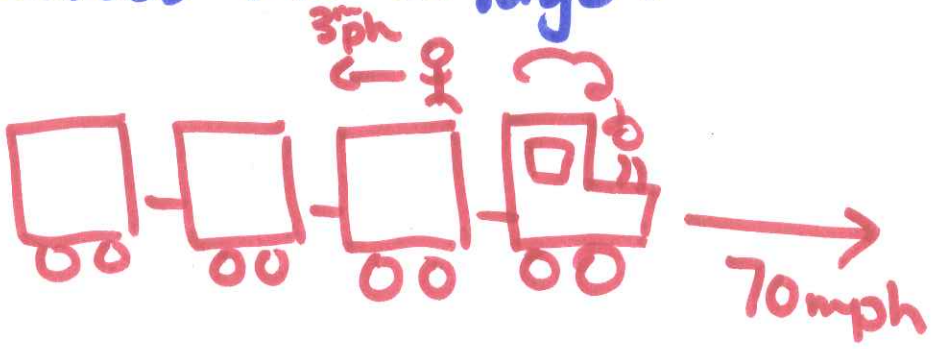
$$\frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$$

$$(25)(400) + (200)\frac{dV}{dt} = 0$$

$$200\frac{dV}{dt} = -10{,}000$$

so $\frac{dV}{dt} = \boxed{-50}$ cm^3/min

Ex: A train is traveling across a bridge at 70 mph. A man on the train is walking toward the rear of the train at 3 mph. How fast is the man traveling across the bridge?

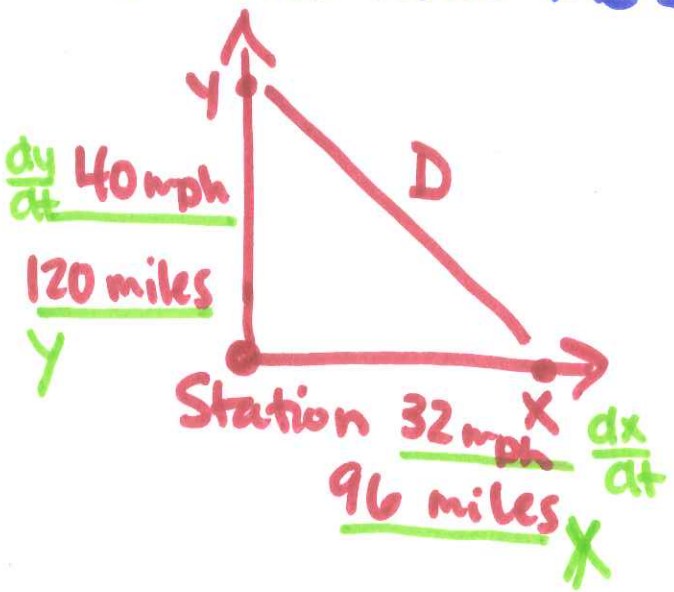


Subtract!

70 - 3 =

67 mph

Ex: 2 trains leave a station at the same time. One travels north on a track at 40 mph. The second travels east on a track at 32 mph. How fast are they traveling away from each other in mph when the northbound train is 120 miles from the station?



find t

$d = r \cdot t$

$120 = 40 \cdot t$

$3 = t$

find x

$d = r \cdot t$

$d = 32 \cdot 3$

$= 96$

$D^2 = x^2 + y^2$

$D^2 = 96^2 + 120^2 = 23616$

so $D = \sqrt{23616}$

$D^2 = x^2 + y^2$

* take derivative with respect to time

$$2D \cdot \underbrace{\frac{dD}{dt}}_{\text{what we want}} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$2\sqrt{23616} \cdot \frac{dD}{dt} = 2(96)(32) + 2(120)(40)$$

$$2\sqrt{23616}\ \frac{dD}{dt} = 15{,}744$$

$$\frac{dD}{dt} = \frac{15744}{2\sqrt{23616}} \approx \boxed{51.22 \text{ mph}}$$